SPATIO-TEMPORAL BEHAVIORS OF CONVECTION IN QUASI-ONE-DIMENSIONAL SYSTEMS

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1.Introduction

Electrohydrodynamic convection (EHC) in nematic liquid crystals is known as a typical system for the study on non-equilibrium pattern dynamics and it has been studied from various view points, for example pattern selection, long wavelength instability, weak turbulence, etc. In this study, we concentrate our attention on the aspect of spatio-temporal behavior in quasi-one-dimensional systems. We are motivated by some experiments on EHC in which coherent oscillating convection state are observed[1,2]. In the system with the aspect ratio for the direction of roll axes is relatively small, two types of oscillation state were found depending on the value of control parameter. Stationary straight rolls become unstable above a critical voltage and roll axes begin to oscillate. That is a coherent oscillation of roll axes which occurs as the secondary instability (OS1). The another one is the phase slip oscillation which appears above the larger value of control parameter (OS2). Furthermore, frequency entrainment phenomena of oscillating convection state under an external field was found[2]. If the relation between a natural frequency and that of an external field is almost rational, mutual entrainment occurs. The oscillation convection in that experiment was OS1 type and domain walls appear depending on the relation between two frequencies. Here we report on computer simulation for a model equation to investigate these non-relaxational spatio-temporal behaviors in quasi-one-dimensional convection systems.

2.Model equations

We can consider various methods of modeling depending on the level of reduction. Models must exhibit non-relaxational feature, because oscillatory convection is time-dependent state. A typical origin that causes non-relaxational behavior is a mean flow effect which is non-local effect. In two dimensional extended EHC, defect turbulence state is realized. In this state, defects are created or annihilated spontaneously due to the mean flow effect. Some model equations which describe such a weak turbulence state have been considered, for example, the anisotropic Swift-Hohenberg equation with a drift term[3] and the simplified amplitude equations[4]. In the later model, one of the oscillatory convection state (OS2) found in experiments was investigated. Here we treat the former model and we expect in the narrow width two-dimensional system, i.e. quasi-one-dimensional system, experimentally observed oscillatory convection state appears. The model equation proposed by Sasa for the description of defect turbulence is[3],

$$\partial_t W + (\vec{U} \cdot \nabla) W = RW - |W|^2 W + \hat{D} W, \qquad (1)$$

$$\hat{D} \equiv -(1+\triangle)^2 - \eta_1 \partial_y{}^4 - 2\eta_2 \partial_x{}^2 \partial_y{}^2, \tag{2}$$

$$\vec{U} = (\partial_y \psi, -\partial_x \psi), \tag{3}$$

$$-\triangle \psi = h\hat{z}(\vec{\nabla} W^* \times \vec{\nabla} \triangle W + c.c.). \tag{4}$$

Here, $\triangle \equiv \partial_x{}^2 + \partial_y{}^2$, W is complex and $Re(W)$ corresponds to the vertical velocity, $\vec{U}$ is the drift field induced by the deformations of the rolls and ψ is the stream function related to vorticity ζ through $\zeta = -\triangle\psi$. A control parameter is R which corresponds to the amplitude of an applied alternating voltage and η_1, η_2 are anisotropic parameters. The parameter h is negative for EHC.

3.Computer simulations

By using the above model equations, we performed computer simulations. Boundary conditions are realistic ones and defined along the boundaries as

$$W = (\vec{n} \cdot \vec{\nabla})W = \psi = 0 \tag{5}$$

where, $\vec{n}$ is the unit vector to the boundary.

Simulations of the equations were performed directly using usual finite difference method. The poisson equation for ψ was solved by SOR method. Parameter values are $\eta_1 = 0.6$, $\eta_2 = 0.3$ and $h = -1.0$. In extended two-dimensional systems, straight normal rolls become unstable at $R_c = -\eta_2/h$ and transits to defect turbulence. Here we used a square lattice $N_x \times N_y$.

Spatial and temporal mesh sizes are $\Delta x = \Delta y = 1.0$ and $\Delta t = 0.02$. We obtained right and left traveling waves instead of oscillatory convections. In Fig.1, a typical example obtained in a numerical simulation is shown. Increasing R, the roll solutions becomes unstable and traveling waves are realized. The direction of propagation depends on the difference scheme. Although we have not yet obtained oscillatory convection, these traveling waves in quasi-one-dimensional systems have also been observed experimentally with a suitable boundary condition.

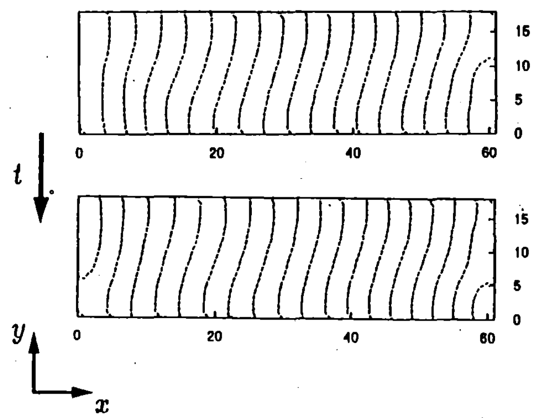

Fig.1 A right traveling wave($R = 1.2$). Solid lines are contours $Re(W) = 0$.

References

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